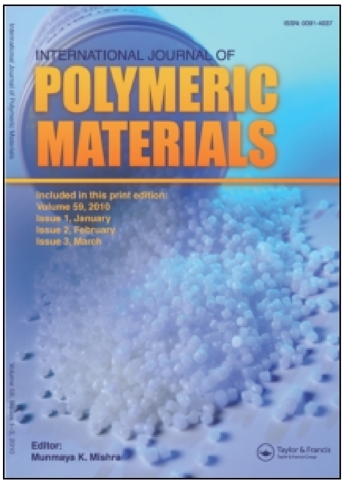

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On the Response of Viscoelastic Materials to Triangle Pulse and Delta Function Excitations

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The stress-strain constitutive relation for linear viscoelastic materials may be expressed on the real time axis by any of several convolution integral formulations. It has been pointed out by Tschoegl[1] that Laplace transformation to the complex plane[2] leads to expressions which are particularly revealing in terms of the physical insight they provide into the relationship between excitation and response. Under such a transformation

$$\bar{\sigma}(s) = s\bar{E}(s)\bar{\epsilon}(s) = \bar{Q}(s)\bar{\epsilon}(s) \tag{1}$$

where σ is the stress response to a strain excitation ϵ, E is the relaxation modulus and Q is the relaxance,[1] an alternative material response function. In Eq. (1) the overstrike indicates the Laplace transformed function $\bar{f}(s)$ of a time dependent function $f(t)$. The dual relation to Eq. (1) involves the creep compliance, D, and the retardance[1,3] function, U.

Equation (1) reveals[1] that the relaxation modulus is the stress response to a unit step function of strain while the relaxance represents the stress response to a unit delta function; this follows from the fact that the Laplace transforms of the unit step and unit delta function are $1/s$ and unity respectively.

Both of the required excitations (delta and step function) are in fact unattainable experimentally. It has been common experimental practice, however, to approximate the step excitation by rapidly stretching a specimen to a predetermined strain and maintaining that strain for all times after the rise time, t_0 (ramp function excitation). While it is clear that the true material parameter $E(t)$ is not obtained if a ramp function excitation is employed, it can be shown[4] that for times significantly greater than t_0, the measured stress response differs only imperceptibly from $E(t)$. This is the often overlooked justification for obtaining stress relaxation data from short rise time ramp excitation experiments. Aklonis and Kelchner[5] have discussed this problem in some detail; they estimate that $10t_0$ is an adequate delay time for stress relaxation experiments on most viscoelastic materials.

Since even the ubiquitous stress relaxation experiment involves some compromise with viscoelastic theory, it is reasonable to inquire into the possibility of an analogous experimental approximation which may allow for direct measurement of the relaxance function. One possible approximation is the unit triangle pulse strain excitation shown in Figure 1 and expressed mathematically as follows

$$T(t) = (1/t_0^2)[th(t) - 2(t - t_0)h(t - t_0) + (t - 2t_0)h(t - 2t_0)] \quad (2)$$

Here $h(t)$ is a unit step function and t_0 is the rise time at which the triangle pulse reaches its peak.

Inserting the Laplace transform of the triangle pulse excitation into Eq. (1) yields

$$\bar{\sigma}(s) = (1/t_0)^2[\bar{E}(s)/s - (2\bar{E}(s)/s)(\exp - st_0) + (\bar{E}(s)/s)(\exp - 2st_0)] \quad (3)$$

The first term can be inverted directly to integral form in the real time plane.[2] The second and third terms can be inverted by employing[6] a transform pair technique. Eq. (3) then becomes

$$\sigma(t, t_0) = (1/t_0)^2[h(t)\int_0^t E(u)\,du - 2h(t - t_0)\int_0^{t-t_0} E(u)\,du + h(t - 2t_0)\int_0^{t-2t_0} E(u)\,du] \quad (4)$$

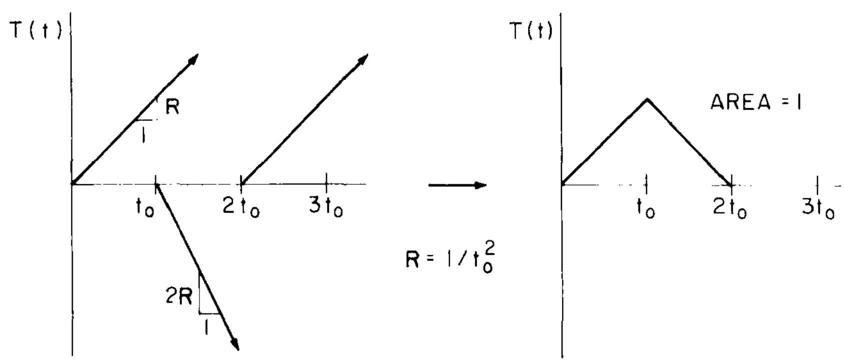

FIGURE 1 Graphical representation of the unit triangle strain pulse, where $R = 1/t_0^2$ is the rate of application and removal of the strain excitation.

Eq. (4) gives the stress response of a viscoelastic material with relaxation modulus $E(t)$ to a unit triangle pulse strain excitation. The response is a family of curves with each time dependent response curve corresponding uniquely to a different excitation rise time t_0.

To illustrate the response of a viscoelastic material, it is necessary to have a viscoelastic model upon which to base the calculations. Often employed for such purposes is the three parameter Maxwell model whose relaxation modulus may be written as

$$E(t) = E_1 + E_2 \exp - t/\tau \tag{5}$$

where τ is the relaxation time, E_1 is the relaxed (equilibrium) modulus and E_2 is the difference between the unrelaxed (glassy) and the relaxed modulus. This expression may be substituted into Eq. (4) to calculate the stress response of the model to a unit triangle strain pulse of arbitrary rise time. Such response curves are shown by the dashed lines in Figure 2 for the various rise times and the model parameters indicated. The solid line in Figure 2 is the relaxance function for the selected model and is obtained by differentiating Eq. (5) with respect to time.

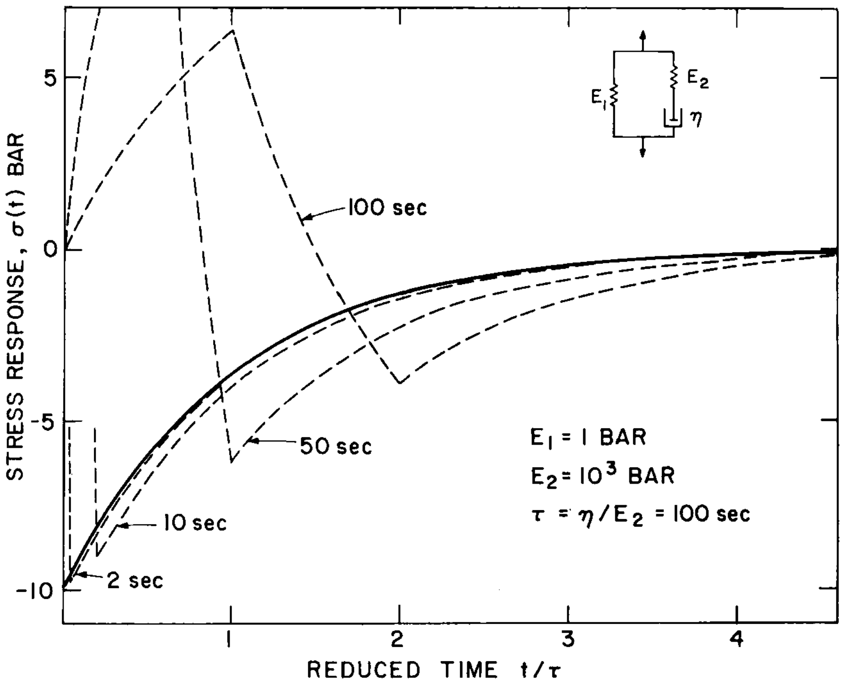

FIGURE 2 Stress response of the three parameter model to a delta function excitation and various triangle pulse excitations. The values of time indicated next to the dashed curves refer to the rise time, t_0, of the corresponding triangle pulse excitation.

Figure 1 demonstrates that the triangle strain pulse may represent a useful method for investigating the relaxance functions of viscoelastic materials. The triangle pulse response curves all approach the relaxance at sufficiently long times; the observed behavior is qualitatively similar to that of ramp function response curves in their approach to the relaxation modulus. The present theoretical analysis says nothing, however, about the feasibility of generating the required triangle pulse strain excitation. Furthermore, stress response data obtained in this way should be treated carefully to exclude inertial contributions and end effects which are bound to complicate such experiments. Even so there is considerable motivation for seeking information on the relaxance function because of the new insight it might provide into the understanding of viscoelastic response. In particular, $Q(t)$ is a direct measure of the first time derivative of the relaxation modulus. This, and the second derivative, $d^2E(t)/dt^2$, provide interesting information[7] on the rate of energy dissipation and the fading memory of viscoelastic materials. The reliability of twice differentiating modulus data is questionable, but if good relaxance data were available, they could be differentiated once with confidence. Thus the triangle pulse experiment and resulting relaxance data would bring us a step closer to obtaining the desired information reliably.

Finally, it may be noted that the triangle pulse is an excitation function of high speed and brief duration. The associated response is, as mentioned above, related to energy dissipation in viscoelastic materials. This suggests that the relaxance may be a material function which correlates with impact resistance in viscoelastic materials. At present none of the linear viscoelastic material functions provides fundamental insight into this important physical property. However, a rise time of less than 0.1 sec would be required for the triangle pulse to approximate impact loading[8] conditions. Such sharp loading of the specimen would certainly result in some "ringing" or related inertial effects which would require great care to overcome in practice.

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